RAMAKRISHNA MISSION VIDYAMANDIRA (Residential Autonomous College affiliated to University of Calcutta) FIRST YEAR [2019-22] B.A./B.Sc. FIRST SEMESTER (July – December) 2019 Mid-Semester Examination, September 2019 * 16/09/2019

Time : 1 pm – 2 pm

Date

MATHEMATICS (Honours) Paper: II [CC 2]

Full Marks : 25

 (2×6)

[Use a separate Answer Book for each group] GROUP – A

Answer **any two** from question nos. 1 to 3 :

1. If the expression $ax^2 + 2hxy + by^2$ transformed to $a'x'^2 + 2h'x'y' + b'y'^2$ by the

transformation of rotation then show that

(i)
$$a' + b' = a + b$$

(ii)
$$a'b' - h'^2 = ab - h^2$$
. [6]

2. If one of the lines of $ax^2 + 2hxy + by^2 = 0$, ab > 0 coincides with one of the lines of

 $a^{\prime}x^{2}+2h^{\prime}xy+b^{\prime}y^{2}=0$ and the remaining two are at right angles, prove that

$$h\left(\frac{1}{b} - \frac{1}{a}\right) = h'\left(\frac{1}{b'} - \frac{1}{a'}\right) = \frac{(a-b)(a'-b')}{2\sqrt{-aa'bb'}}.$$
[6]

3. A conic $\frac{l}{r} = 1 + e \cos \theta$ is cut by a circle passing through the pole in four points whose radius

vectors are r_1 , r_2 , r_3 , r_4 . Show that

$$\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4} = \frac{2}{l} \quad and \quad r_1 r_2 r_3 r_4 = \frac{4d^2 l^2}{e^2}.$$
 [6]

<u>GROUP – B</u>

4. Answer any two :---

a) Solve by the method of variation of parameters:

$$x \log x \frac{dy}{dx} + y = 2 \log x.$$

b) Obtain the general solution and singular solution (if any) of the equation

$$y + px = p^2 x^4$$
, where $p = \frac{dy}{dx}$.

c) Solve:
$$3ydx - 2xdy + x^2y^{-1}(10ydx - 6xdy) = 0.$$

- 5. Answer any one :--
 - a) Show that the equation $(x^3 3x^2y + 2xy^2)dx (x^3 2x^2y + y^3)dy = 0$ is exact and find the solution if y = 1 when x = 1.
 - b) Obtain the differential equation of the family of curves $\frac{x^2}{a^2 + \lambda} + \frac{y^2}{b^2 + \lambda} = 1$, where a,b are fixed constants and λ is an arbitrary parameter.

(1)

 (1×3)

 (2×5)